P.5 Rational Expressions

fraction

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Domain: The set of real numbers for which an algebraic expression is defined (the input).

↑ ?

- divide by zero
- square root of neg.

Domain?

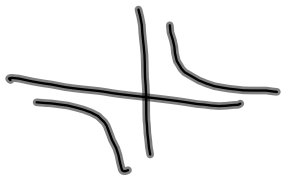
a) $x^4 + 3x^3 - x + 5$ all $\mathbb{R}$

b) $\sqrt{x - 3}$ all $\mathbb{R}$, $x \geq 3$

c) $\frac{x + 3}{x - 3}$ all $\mathbb{R}$, $x \neq 3$

Rational expression: The quotient of two polynomials

$y = \frac{1}{x}$



$$\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$$

$$\frac{(2x+4)}{8} \overset{\text{NO!}}{=}$$

$$\frac{2(x+2)}{4 \cdot 8}$$

Simplifying rational expressions:

$$\frac{x^2 + 4x - 12}{3x - 6} = \frac{(x+6)(x-2)}{3(x-2)}$$

"hole"

$3x+6$

$$= \frac{x+6}{3}$$

$$= \frac{1}{3}x + 2$$

$$\frac{12 + x - x^2}{2x^2 - 9x + 4} = \frac{(4 - x)(3 + x)}{(2x - 1)(x - 4)}$$

$4 - x \stackrel{?}{=} x - 4$

$-1(1 - x) = -4 + x$

$= x - 4$

$x + 4 = -(x - 4)$

$\frac{x+4}{x-4} = -1$? NO

$$= \frac{-3 - x}{2x - 1}$$

$$\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \cdot \frac{\overset{5}{\cancel{25}}}{\cancel{9}_{3}} = \frac{75}{45} = \frac{5}{3}$$

$$\frac{5}{3}$$

$$\frac{3}{4} \div \frac{27}{14}$$ flip

$$\frac{3}{\cancel{4}} \cdot \frac{\cancel{14}^{7}}{\cancel{27}_{9}} = \frac{7}{9}$$

Multiply:

$$\frac{x^2 - x - 6}{x^2 + 6x + 9} \cdot \frac{x + 3}{x^2 - 4}$$

$$\frac{(x-3)(x+2)}{(x+3)(x+3)} \cdot \frac{x+3}{(x+2)(x-2)}$$

$$= \frac{x-3}{(x+3)(x-2)} = \frac{x-3}{x^2+x-6}$$

Divide:

$$\frac{x^2 - 14x + 49}{x^2 - 49} \div \frac{3x - 21}{x + 7}$$

$$\frac{(x-7)(x-7)}{(x-7)(x+7)} \cdot \frac{x+7}{3(x-7)}$$

$$= \frac{1}{3}$$

$$\frac{1 \cdot 3}{5 \cdot 3} + \frac{2 \cdot 5}{3 \cdot 5} - \frac{7}{15}$$
$$3 \cdot 5$$

$$\frac{3 + 10 - 7}{3 \cdot 5} = \frac{6}{15}$$

Subtract:

$$\frac{2}{x^2-4} - \frac{1}{x^2+3x+2}$$

$(x-2)(x+2)$ $(x+1)$ $(x+2)(x+1)$ $(x-2)$

$$\frac{2x+2 \quad -x \quad +2}{(x-2)(x+2)(x+1)}$$

$$= \frac{x+4}{(x-2)(x+2)(x+1)}$$

Add:

$$\frac{-1}{x} + \frac{2}{x^2+1} + \frac{1}{x^3+x}$$

(x^2+1) (x) $x(x^2+1)$ ← factor

$$\frac{-x^2-1+2x+1}{x(x^2+1)}$$ ← common denominator

$$= \frac{-x^2+2x}{x(x^2+1)} = \frac{x(-x+2)}{x(x^2+1)}$$

$$= \frac{-x+2}{x^2+1}$$

Simplify.

$$\frac{x^2 - 4}{x(x - 2) - 7(x - 2)}$$

$$= \frac{(x-2)(x+2)}{(x-2)(x-7)}$$

- a. $\frac{x + 2}{x - 7}$
- b. $\frac{x - 2}{x - 7}$
- c. $\frac{x - 2}{x + 7}$
- d. $\frac{x + 2}{x + 7}$

Multiply.

$$\frac{8x^2 - 10x}{56x - 64} \cdot \frac{7x - 8}{10x - 12}$$

$$= \frac{2 \cdot x(4x-5)}{8(7x-8)} \cdot \frac{7x-8}{2(5x-6)}$$

$$= \frac{x(4x-5)}{8(5x-6)}$$

- a. $\frac{4x - 5}{5x - 6}$
- b. $\frac{x(4x - 5)}{8(5x - 6)}$
- c. $\frac{x(7x - 8)}{8(5x - 6)}$
- d. None of the above.