

P.5 Rational  
Expressions

*fraction*  
↗



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Domain: The set of real numbers for which an algebraic expression is defined (the input).



- divide by zero
- square root of neg.

Domain?

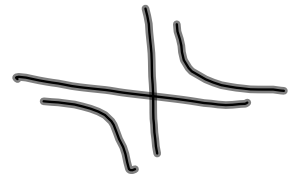
a)  $x^4 + 3x^3 - x + 5$      all  $\mathbb{R}$

b)  $\sqrt{x - 3}$      all  $\mathbb{R}$ ,  $x \geq 3$

c)  $\frac{x + 3}{x - 3}$      all  $\mathbb{R}$ ,  $x \neq 3$

Rational expression: The quotient of two polynomials

$$y = \frac{1}{x}$$



$$\frac{16}{24} = \frac{2 \cdot \cancel{8}}{3 \cdot \cancel{8}} = \frac{2}{3}$$

$(\underline{2x + 4})$  = No!  
 $\downarrow$   
 $\frac{\cancel{2}(x+2)}{4\cancel{2}}$

Simplifying rational expressions:

$$\begin{aligned} \frac{(x^2 + 4x - 12)}{(3x - 6)} &= \frac{(x+6)(x-2)}{3(x-2)} \quad \text{"holt"} \\ &= \frac{x+6}{3} \\ &= \frac{1}{3}x + 2 \end{aligned}$$

*Handwritten notes:*  
- The numerator  $(x^2 + 4x - 12)$  is underlined in yellow.  
- The denominator  $(3x - 6)$  is underlined in green.  
- The term  $3x+2$  is written below the denominator and underlined in green.  
- The term  $(x-2)$  in the denominator of the second step is circled in green.  
- The term  $x+6$  in the numerator of the second step is underlined in red.

$$\frac{12 + x - x^2}{2x^2 - 9x + 4} = \frac{(\overset{-1}{\cancel{4}} - x)(3 + x)}{(2x - 1)(\cancel{x - 4})}$$

$$4 - x \stackrel{?}{=} x - 4$$

$$-1(-x) = -4 + x$$

$$= x - 4$$


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$$x + 4 = -(x - 4)$$

$$\frac{x + 4}{x - 4} = -1? \quad \underline{\underline{NO}}$$

$$= \frac{-3 - x}{2x - 1}$$



$$\frac{\overset{1}{\cancel{3}}}{\cancel{5}} \cdot \frac{\overset{5}{\cancel{25}}}{\cancel{A}_3} = \frac{75}{45} = \frac{5}{3}$$

$$\left( \frac{5}{3} \right)$$

$$\frac{3}{4} \div \frac{27}{14} \quad \text{flip}$$

$$\frac{\cancel{3}}{\cancel{4}} \cdot \frac{\cancel{14}^4}{\cancel{27}_9} = \frac{4}{9}$$

Multiply:

$$\frac{x^2 - x - 6}{x^2 + 6x + 9} \cdot \frac{x + 3}{x^2 - 4}$$

$$\begin{aligned} & \frac{(x-3)(\cancel{x+2})}{(\cancel{x+3})(x+3)} \cdot \frac{\cancel{x+3}}{(\cancel{x+2})(x-2)} \\ & = \frac{x-3}{(x+3)(x-2)} = \frac{x-3}{x^2+x-6} \end{aligned}$$

Divide:

$$\frac{x^2 - 14x + 49}{x^2 - 49} \div \frac{3x - 21}{x + 7}$$

$$\frac{(x-7)(x-7)}{(x-7)(x+7)} \cdot \frac{x+7}{3(x-7)}$$

$$= \frac{1}{3}$$

$$\frac{1 \cdot 3}{5 \cdot 3} + \frac{2 \cdot 5}{3 \cdot 5} - \frac{7}{15}$$

$3 \cdot 5$

$$\frac{3 + 10 - 7}{3 \cdot 5} = \frac{6}{15}$$

Subtract:

$$\frac{2 \overset{(x+1)}{\cancel{-}}}{x^2 - 4} - \frac{1 \overset{(x-2)}{\cancel{-}}}{x^2 + 3x + 2}$$

$(x-2)(x+2)$   $(x+1)$   $(x+2)(x+1)(x-2)$

$$\frac{2x + 2 \quad -x \quad + 2}{(x-2)(x+2)(x+1)}$$

$$= \frac{x + 4}{(x-2)(x+2)(x+1)}$$

Add:

$$\frac{-1}{x} + \frac{2}{x^2+1} + \frac{1}{x^3+x}$$

*(x<sup>2</sup>+1)* *(x)* *(x<sup>2</sup>+1)* *x(x<sup>2</sup>+1)* ← factor

$$\frac{-x^2 - 1 + 2x + 1}{x(x^2+1)}$$

← common denominator

$$= \frac{-x^2 + 2x}{x(x^2+1)} = \frac{x(-x+2)}{x(x^2+1)}$$

$$= \frac{-x+2}{x^2+1}$$

Simplify.

$$\frac{x^2 - 4}{x(x - 2) - 7(x - 2)}$$

$$= \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-7)}$$

a.  $\frac{x + 2}{x - 7}$

b.  $\frac{x - 2}{x - 7}$

c.  $\frac{x - 2}{x + 7}$

d.  $\frac{x + 2}{x + 7}$

Multiply.

$$\frac{8x^2 - 10x}{56x - 64} \cdot \frac{7x - 8}{10x - 12}$$

$$\begin{aligned} & \frac{\cancel{2}x(4x-5)}{\cancel{8}(7x-8)} \cdot \frac{\cancel{7x-8}}{\cancel{2}(5x-6)} \\ & = \frac{x(4x-5)}{8(5x-6)} \end{aligned}$$

- a.  $\frac{4x - 5}{5x - 6}$
- b.  $\frac{x(4x - 5)}{8(5x - 6)}$
- c.  $\frac{x(7x - 8)}{8(5x - 6)}$
- d. None of the above.



